AD-A008 473

A SIMPLIFIED METHOD FOR ASSESSING THE PERFORMANCE OF AN ENGINE IN THE ACCELERATION ROLE

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February 1975

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFAPL-TR-74-73
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER: AD-A008 423
4. TITLE (and Subtitle): A Simplified Method for Assessing the Performance of an Engine in the Acceleration Role
5. TYPE OF REPORT & PERIOD COVERED: Final, 1969-1973
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Edward T. Curran, M. Brian Bergsten
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: AFAPL/RJT, WPAFB, Ohio 45433
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 30121201
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Aero Propulsion Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433
12. REPORT DATE: February 1975
13. NUMBER OF PAGES: 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS (of this report): UNCLASSIFIED
15a. DECLASSIFICATION DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Engine-Vehicle Performance
Flight Mechanics
Boost Engine

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report gives a simple analysis for assesssing the relative importance of engine specific weight and specific fuel consumption for an acceleration engine. Such an engine may power the boost phase of a single-stage-to-orbit vehicle, the first stage of a multistage vehicle, or the acceleration phase of a hypersonic cruise vehicle; it is assumed that the performance of later stages is not relevant to the optimization. The major limitations of the method are the assumptions that the thrust/drag ratio and the specific fuel

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Item 20 Continued - Abstract

consumption are constant during the flight and the fact that no restriction is put on vehicle acceleration. However, within these limitations the analysis allows comparisons to be made over a wide range of engine types.

FOREWORD

This report is an unclassified and more general version of a Technical Memorandum entitled "A Simplified Method of Comparing the Performance of Booster Powerplants for Aerospace Vehicles" authored by E.T. Curran, M.B. Bergsten, and J.R. Smith. This work was performed under Project 3012, "Ramjet Technology", Task 301211, "Ramjet Design and Assessment", Work Unit Number 30121103, during the period January 1973 to March 1973.

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LIST OF ILLUSTRATIONS

SYMBOLS

B	drag/weight
C	constant of flight path
D	drag force
G	payload function = $\frac{P + k_1 + k_2}{q}$
h	altitude
I_{sp}	fuel specific impulse
$I_{sp_{eff}}$	effective fuel specific impulse $I_{sp_{eff}} = I_{sp} \frac{T-D}{T}$
k_1	constant in structural equation $M_s = k_1M_1 + K_2M_f$
k_2	constant in structural equation $M_s = k_1M_1 + k_2M_f$
M	mass
M_1	take-off mass of vehicle
M_2	mass of vehicle at end of boost phase: in case of multistage vehicle this is before separation
M_3	mass of vehicle after separation from first stage
M_a	mass of vehicle at start of trajectory defined by $\rho V^n = C$
M_{be}	mass of boost engines
M_{bf}	mass of fuel used in boost phase
M_f	total mass of fuel
$\dot{M}_f$	rate of fuel flow
M_{orb}	total mass of vehicle in orbit
M_p	effective payload = $M_{orb} - M_s - M_{be} - M_{se}$
M_s	mass of vehicle structure
M_{se}	mass of sustainer engines used in single-stage to orbit vehicle
n	index in trajectory equation $\rho V^n = C$

P	payload ratio $\frac{M_p}{M_1}$ for single-stage to orbit vehicle
P_m	payload ratio $\frac{M_s}{M_1}$ for multistage to orbit vehicle
q	parameter of mass ratios
q_1	parameter of mass ratios applied to single-stage to orbit vehicle $= \frac{M_{orb}}{M_2}(1 + k_2) - \frac{M_{se}}{M_2}$
q_m	parameter of mass ratios applied to multistage vehicle $= 1 + k_2$
S	plan area of aircraft
SW	specific weight of engine
s.f.c.	specific fuel consumption of engine
t	time
v	volume of aircraft
V	velocity
V_1	initial velocity
V_a	velocity at start of trajectory defined by $\rho V^n = C$
W	weight
W_1	take-off weight of vehicle
X_s	pylon (or stub) thrust
X_{s_1}	initial stub thrust
α	$\frac{\text{thrust}}{\text{drag}}$, $\frac{X_s}{D} = \frac{1}{B}\frac{X_s}{W}$
β	Constant of density - height relationship
$(\Delta V)\varepsilon$	effective velocity increment $= \Delta V\left[1 + \frac{ng}{\beta V_1 (V_1 + \Delta V)}\right]$
θ	angle of flight path
ρ	air density
ρ_o	air density at sea level

SECTION I
INTRODUCTION

When considering engines for an aircraft whose major requirement is for long-range or long-duration, it is clear that the overall propulsive efficiency is of paramount importance. The engine weight is usually much lower than the fuel weight, and therefore the engine specific weight and specific thrust (while by no means unimportant) can be considered less significant than specific fuel consumption at the cruise condition.

In vehicles of shorter range (where the acceleration phase becomes more important) this position is substantially changed. In particular, where a high acceleration is required, as for example in interceptor missiles, a high installed engine thrust greatly increases the importance of low engine specific weight. The relative unimportance of low fuel consumption, i.e., of high engine efficiency, makes it possible to consider trends in engine design which may sacrifice some measure of efficiency while reducing engine weight.

The object of this report* is to derive a simple yardstick by means of which the relative merits of different engine systems can be compared. This is attempted in the context of a simple acceleration mission; that of a recoverable first stage of an ortital vehicle. In this case, it is not apparent whether the class of low s.f.c. but high specific weight engines such as the turbojet is superior to the higher s.f.c. but lower weight engines typified by the liquid propellent rocket. In addition, the method evolved allows the relative influences of such parameters as the lift/drag ratio and the structural weight fraction of the vehicle to be quantitatively assessed.

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*This report is an unclassified and more general version of Reference 1 which was an in-house report which received limited distribution. Various similar analyses have now appeared in the open literature (References 2 and 3).

Two classes of vehicle are considered: the single-stage to orbit vehicle and the multistage vehicle. For the single-stage vehicle, it is considered that the vehicle is boosted (by airbreathing engines) up to a given boost velocity at which these engines are shut down: unspecified "sustainer" engines are then used to accelerate the vehicle to orbital conditions. For the multistage vehicle, it will be assumed that the complete vehicle is accelerated by these airbreathing engines up to the first-stage separation velocity. For both types of vehicle the merit of a given engine is assessed on the basis of an effective payload fraction delivered into orbit.

For a complete comparison of the various classes of boost engine, it is necessary to ensure that each engine class is installed and operated in such a manner as to maximize the payload fraction. Ideally, this would involve optimization of the engine/vehicle characteristics and the flight profile for each class of engine. However, for the purpose of this analysis the flight path and vehicle aerodynamics are postulated and the only optimization attempted is with respect to the thrust/weight ratio of the vehicle.

It must be emphasized that this analysis is limited to considerations of weight changes. No attempt is made to examine the equally important criterion of cost, though it may be noted that within any narrow class of engines, it seems likely that low cost, not only of manufacture but also of research and development, will be associated with simplicity; this in turn may be associated with a lower weight and a lower efficiency than could be achieved with maximum resort to complexity.

SECTION II
OPTIMIZATION OF THE EFFECTIVE PAYLOAD FRACTION

1. Single-Stage-to-Orbit Vehicle

For a single-stage-to-orbit vehicle the effective payload is defined as

$$M_p = M_{orb} - M_s - M_{be} - M_{se} \tag{1}$$

where

M_{orb} is the mass of the vehicle in orbit
M_s is the mass of the vehicle structure
M_{be} is the mass of the boost engines
M_{se} is the mass of the sustainer engines

Obviously, this definition of the effective payload includes many items, such as vehicle systems, which are not normally counted as payload. However, for the purposes of this analysis, the effective payload is considered to be a suitable index of relative performance.

The mass of the vehicle structure is assumed to consist of two components: one component is proportional to the initial mass of the vehicle, the other to the total mass of fuel. Thus,

$$M_s = k_1 M_1 + k_2 M_f \tag{2}$$

where

M_1 is the initial mass of the vehicle
M_f is the total mass of fuel
k_1, k_2 are constants.

From Equations 1 and 2 the effective payload fraction can be written

$$\frac{M_p}{M_1} = \left(\frac{M_{orb}}{M_2}(1 + k_2) - \frac{M_{se}}{M_2}\right)\frac{M_2}{M_1} - (k_1 + k_2) - \frac{M_{be}}{M_1}$$

or

$$\frac{M_p}{M_1} = q_1 \frac{M_2}{M_1} - (k_1 + k_2) - \frac{M_{be}}{M_1} \tag{3}$$

where

$$q_1 = \frac{M_{orb}}{M_2}(1 + k_2) - \frac{M_{se}}{M_2} \tag{4}$$

$$\frac{P_1 + k_1 + k_2}{q_1} = \frac{M_2}{M_1} - \frac{1}{q_1}\frac{M_{be}}{M_1}$$

or, in terms of the specific weight of the boost engine, SW,

$$\frac{P_1 + k_1 + k_2}{q_1} = \frac{M_2}{M_1} - \frac{SW}{q_1}\left(\frac{X_{s_1}}{W_1}\right) \tag{5}$$

where X_{s_1}, W_1 are the initial values of "pylon" thrust and vehicle weight. It will be assumed that the mass ratios $\frac{M_{orb}}{M_2}$ and $\frac{M_{se}}{M_2}$ corresponding to the sustainer phase are fixed, so that q_1 is constant. Then the maximum value of P_1 occurs at the maximum value of the R.H.S. of Equation 5.

2. Multistage to Orbit Vehicle

For the multistage vehicle, it will be assumed that the separation velocity of the first stage and mass ratios of subsequent stages are fixed, and that the structure weight of the first stage obeys an equation similar to Equation 2. Maximum payload fraction will occur when the mass ratio after the first-stage separation, $\frac{M_3}{M_1}$, is a maximum. Then

$$\frac{M_3}{M_1} = \frac{M_1 - (M_{bf} + M_{be} + M_s)_{first\ stage}}{M_1}$$

where M_{bf} is the fuel used by the first (boost) stage

$$(M_s)_{first\ stage} = k_1 M_1 + k_2 M_{bf}$$

and it should be noted that the value of k_1 used here in not necessarily identical with that appropriate to Equation 2.

If we set

$$q_m = 1 + k_2 \tag{6}$$

(a form identical to Equation 4 if there is no sustainer phase in the single-stage case) we have

$$\frac{M_3}{M_1} = q_m \frac{M_2}{M_1} - (k_1 + k_2) - \frac{M_{be}}{M_1} \tag{7}$$

or, writing P_m for $\frac{M_3}{M_1}$,

$$\frac{P_m + k_1 + k_2}{q_m} = \frac{M_2}{M_1} - \frac{SW}{q_m}\left(\frac{X_{s_1}}{W_1}\right) \tag{8}$$

Comparing this with Equation 5 we see that for either the single or multistage vehicles the maximum effective payload fraction is obtained when

$$\frac{\partial}{\partial\left(\frac{X_{s_1}}{W_1}\right)}\left(\frac{M_2}{M_1} - \frac{SW}{q}\ \frac{X_{s_1}}{W_1}\right) = 0$$

i.e., when

$$\frac{\partial\left(\frac{M_2}{M_1}\right)}{\partial\left(\frac{X_{s_1}}{W_1}\right)} = \frac{SW}{q} \tag{9}$$

where

$q = q_1$ for single-stage vehicle

$q = q_m$ for multistage vehicle.

3. Flight Path Analysis

An expression must now be sought for the derivative in Equation 9 from the flight mechanics of the vehicle.

For a vehicle in climbing flight we have the following expression

$$M\frac{dV}{dt} = X_s - D - M_g \sin\theta \tag{10}$$

where

M is the vehicle mass

X_s is the "pylon" thrust

D is the drag

V is the velocity

θ is the angle of climb

It will be assumed that the vehicle accelerates along a trajectory defined by

$$\rho V^n = C \tag{11}$$

where

ρ is air density

n and C are constants,

and also that the density-height relationship for the atmosphere is given by

$$\rho = \rho_o e^{-\beta h} \tag{12}$$

where

h is altitude

ρ_o is density at sea level

β is a constant.

Now the flight path angle may be expressed by

$$\sin\theta = \frac{1}{V}\frac{dh}{dt}$$

and from Equations 11 and 12

$$\frac{dh}{dt} = \frac{n}{V\beta}\frac{dV}{dt}$$

so that

$$\sin\theta = \frac{n}{\beta V^2}\frac{dV}{dt} \tag{13}$$

Then, from Equations 10 and 13,

$$\left(\frac{1}{g} + \frac{n}{\beta V^2}\right) \frac{dV}{dt} = \frac{X_s - D}{Mg} \tag{14}$$

Introducing s.f.c. = $\frac{3600\ g\ M_f}{X_s}$ and remembering that M decreases during flight we have

$$\left(\frac{1}{g} + \frac{n}{\beta V^2}\right) \frac{dV}{dt} = - \frac{3600}{\text{s.f.c.}} \left(1 - \frac{D}{X_s}\right) \frac{1}{M} \frac{dM}{dt} \tag{15}$$

This equation can be integrated if we assume that g, s.f.c., and the ratio $\frac{D}{X_s}$ are constant during the flight. Taking V_1 as the velocity at the start of acceleration and ΔV as the velocity increment we have

$$\frac{\Delta V}{g} \left[1 + \frac{ng}{\beta V_1 (V_1 + \Delta V)}\right] = \frac{3600}{\text{s.f.c.}} \left(1 - \frac{D}{X_s}\right) \ln \frac{M_1}{M_2} \tag{16}$$

The L.H.S. of Equation 16 may be conveniently written as $\frac{(\Delta V)_\varepsilon}{g}$ where by $(\Delta V)_\varepsilon$ we understand the effective velocity increment, which is greater than the true velocity increment by a factor that takes into account the climbing flight of the vehicle. Finally, we express

$$\frac{D}{X_s} = \frac{B}{X_s/W}$$

where $B = \frac{D}{W}$ is assumed constant, implying also that X_s/W is constant. Equation 16 then becomes:

$$\frac{(\Delta V)_\epsilon}{g} = \frac{3600}{\text{s.f.c.}} \left(1 - \frac{B}{X_s/W}\right) \ln \frac{M_1}{M_2} \tag{17}$$

We may remark that the acceleration phase of the trajectory defined by Equation 11 might start at conditions other than those of take-off; in this case V_1 should be replaced by V_a and M_1 by M_a where the suffix

a refers to conditions at the start of the climbing phase. In general the mass ratio corresponding to take-off and transition to a ρV^n trajectory is not explicitly considered in this analysis.

4. Generalized Optimum Solution

We may now obtain from Equation 17 the derivative which occurs in Equation 9; thus

$$\frac{\partial\left(\frac{M_2}{M_1}\right)}{\partial\left(\frac{X_{s_1}}{W_1}\right)} = -\frac{\partial\left(\frac{M_2}{M_1}\right)}{\partial\left(\frac{X_s}{W}\right)} = \left[\frac{-B\left(\frac{X_s}{W}\right)^{-2}}{1-B\left(\frac{X_s}{W}\right)^{-1}}\right]\frac{M_2}{M_1}\ln\frac{M_2}{M_1} \tag{18}$$

Substituting this result in Equation 9 we see that the optimum thrust/weight ratio of the vehicle (i.e., that giving the maximum payload fraction) is given by

$$\frac{SW}{q} = \left[\frac{-B\left(\frac{X_s}{W}\right)^{-2}}{1-B\left(\frac{X_s}{W}\right)^{-1}}\right]\frac{M_2}{M_1}\ln\frac{M_2}{M_1} \tag{19}$$

Now Equations 5 and 8 may be written generally as

$$G = \frac{M_2}{M_1} - \frac{SW}{q}\left(\frac{X_s}{W}\right)$$

where

$$G = \frac{P + k_1 + k_2}{q}$$

Combining this with Equation 19 we have

$$\frac{1}{\alpha - 1}\ln\frac{M_1}{M_2} + G\frac{M_1}{M_2} = 1 \tag{20}$$

where

$$\alpha = \frac{1}{B}\frac{X_s}{W} = \frac{X_s}{D}$$

Finally, we may express the optimum values of engine specific weight and specific fuel consumption in terms of the groups $\frac{B}{q}\cdot SW$ and $\frac{(\Delta V)_\epsilon}{g}\;\frac{s.f.c.}{3600}$

where

$$\frac{B}{q}\cdot SW = \left(\frac{1}{\alpha}\right)\left(\frac{M_2}{M_1} - G\right) \tag{21}$$

and

$$\frac{(\Delta V)_\epsilon}{g}\;\frac{s.f.c.}{3600} = \left(1 - \frac{1}{\alpha}\right)\ ln\ \frac{M_1}{M_2} \tag{22}$$

from Equations 17, 19, and 20.

SECTION III
DISCUSSION

The optimization process outlined in the body of this report may be made clearer by a simple graphical illustration. If a multistage vehicle is considered with a coefficient k_2 equal to zero, then the parameter q_m is equal to unity. We see from Equation 8 that maximum payload fraction occurs when $\frac{M_2}{M_1} - SW\left(\frac{X_{s_1}}{W_1}\right)$ takes its maximum value, i.e., when

$$\left(\frac{M_{bf}}{M_1} + SW\frac{X_{s_1}}{W_1}\right) \text{ is a minimum.}$$

Thus, in this example the optimization reduces simply to minimizing the sum of the engine plus fuel fractions. This process is illustrated in Figure 1. As the thrust/weight ratio of the vehicle is increased, then, for constant engine specific weight, the engine mass fraction $\frac{M_{be}}{M_1}$ must increase linearly. Also for constant s.f.c., $\frac{M_2}{M_1}$ increases with vehicle thrust/weight ratio, as may be seen from Equation 17 when $(\Delta V)_\varepsilon$ is fixed and $B = \frac{D}{W}$ is assumed constant: correspondingly the value of $\frac{M_{bf}}{M_1}$ decreases. From Figure 1, it is apparent that the optimum thrust/weight ratio corresponds to the minimum value of $\frac{M_{bf}}{M_1} + \frac{M_{be}}{M_1}$.

For this simple illustration we see from Equation 9 that at optimum conditions the slope of the lower curve is equal to that of the upper line,

$$\text{i.e.,} \qquad \frac{\partial\left(\frac{M_2}{M_1}\right)}{\partial\left(\frac{X_s}{W}\right)} = SW.$$

Figure 2 shows plots of the generalized solution resulting from Equations 20, 21, and 22. For ease of presentation the term $\frac{(\Delta V)_E}{g} \frac{s.f.c.}{3600}$ has been replaced by $\frac{10^4}{(\Delta V)_E}$ x s.f.c. and this latter quantity is plotted against $\frac{B}{q}$ SW for fixed values of $G = \frac{P_1 + k_1 + k_2}{q}$, and for fixed α, i.e., fixed thrust/drag ratio at optimum conditions.

We can make some general deductions from Figure 2. For given payload and effective velocity increment, an increase in specific weight requires a decrease in s.f.c.; the greater the payload the greater the decrease in s.f.c. that would be required. For a given thrust/drag ratio there is a maximum allowable engine specific weight; the value of $\frac{(\Delta V)_E}{g} \frac{s.f.c.}{3600}$ associated with this can be shown to be $\left(1 - \frac{1}{\alpha}\right)$ and the corresponding value of $\frac{B}{q}$ SW is $\frac{1}{\alpha\,(\alpha - 1)} \cdot \frac{1}{e}$.

Now, for further illustration, consider a multistage vehicle with the following characteristics:

B = drag/weight = 0.5

$k_1 = 0.3$

$k_2 = 0.$

Then $M_s = k_1 M_1 + k_2 M_{bf} = 0.3 M_1$ and $q_m = 1 + k_2 = 1$. It will be assumed that the vehicle flies on a constant ρV^2 trajectory over a velocity range defined by V_a = 760 ft/s, ΔV = 3000 ft/s. If $\beta = 3.383 \times 10^{-5}$ ft^{-1}, the effective velocity increment (Equation 16) is 5000 ft/s. These data and Figure 2 are used to plot specific fuel consumption against specific weight for constant payload fraction, given by $P = G - k_1$, in Figure 3. We see that a liquid propellant rocket engine with a specific fuel consumption of nine and a specific weight of 0.01 gives a value of P a little in excess of 0.3. An airbreathing engine with an s.f.c. of four will break even with the rocket engine at a specific weight of about 0.12. The structural constants k_1, k_2 are assumed the same for both rocket and airbreathing systems. The initial acceleration related to the values of s.f.c. and specific weight is shown in Figure 2 by curves derived from Equation 26 and the lines of constant α. Thus,

in the oversimplified comparison just quoted, the high s.f.c. rocket engine would require an initial acceleration of about 0.8 g while the low s.f.c. airbreathing engine has an initial acceleration of only 0.17 g; i.e., the engine with lower specific weight will be used with higher installed thrust/weight ratios to keep down the time of flight and hence, total fuel consumption.

Examples such as this suggest the rather loose statement that for engines with high s.f.c. and low specific weight, such as rockets, the most effective way of improving the payload potential of the engine is to reduce the s.f.c. For engines with low s.f.c. and high specific weight, such as turbojets, the best prospects lie in reducing the engine specific weight. A detailed study of the performance and specific weight of power plants intermediate between the rocket and turbojet extremes may yield an acceleration engine offering greater payload potential than its progenitors.

The data presented in Figure 2 permit many quantitative studies to be made of the effect on the payload potential of a vehicle, of varying engine, vehicle, and flight path parameters. Various refinements may be added to the analysis. For example, it is not necessary to restrict the trajectory to one constant ρV^n arc: several arcs of this form may be used to approximate to a given trajectory.

Another presentation of the data given in Figure 2 is informative. So far the analysis has been expressed in terms of conventional aircraft engine terms, such as specific fuel consumption (s.f.c.) and specific weight (SW). In rocket engine practice the comparable terms fuel specific impulse (I_{sp}) and engine thrust-to weight ratio (τ) are used.

Equations 21 and 22 may be rearranged in terms of I_{sp} and τ to yield

$$\frac{B}{q}\frac{1}{\tau} = \frac{1}{\alpha}\left(\frac{M_2}{M_1}\right) - G \qquad (23)$$

and

$$\frac{(\Delta V)_\epsilon}{g} = I_{sp}\left(1 - \frac{1}{\alpha}\right) \ln \frac{M_1}{M_2} \qquad (24)$$

one may write

$$I_{sp}\left(1 - \frac{1}{\alpha}\right) = I_{sp}\left(\frac{X_s - D}{X_s}\right) = I_{sp_{eff}}$$

where $I_{sp_{eff}}$ is termed the effective specific impulse. Now Equation 24 can be expressed as

$$\frac{(\Delta V)_\epsilon}{g} = I_{sp_{eff}} \ln \frac{M_1}{M_2} \qquad (25)$$

which is directly comparable to the classical rocket equation relating mass ratio to velocity change.

Thus, the engine can be considered to operate at an effective specific impulse which is less than the potentially available specific impulse due to the factor $\frac{X_{s-D}}{X_s}$, or $1-\frac{D}{X_s}$. The ratio D/X_s cannot be reduced arbitrarily because beyond a certain point more thrust implies more engine weight and therefore, less payload. Now a graph equivalent to Figure 2 can be plotted using Equations 23 and 25 as shown in Figure 4. It is apparent that only engines of high thrust-to-weight ratio are able to operate effectively at values of $I_{sp_{eff}}$ near their basic impulse. For turbo-machine systems of low thrust-to-weight ratio, it is apparent that the high basic impulse of these engines cannot be realized in the acceleration role. Another presentation of these relationships is shown in Figure 5. Again the significant tradeoff between engine thrust-to-weight ratio and specific impulse is portrayed.

The fuel density can exert a marked effect on the vehicle structural weight and aerodynamics. Equation 2 can be modified so that these effects can be studied. Thus the expression

$$M_s = k_1 M_1 + k_2 M_f$$

may be written

$$M_s = k_1 M_1 + \left(k_2 \rho f \frac{v_f}{v}\right) v$$

where v denotes volumes, thereby relating the structural mass to the fuel density, the utilization of the available volume for fuel stowage, and the overall volume of the vehicle. The aircraft volume can be linked to the aerodynamic performance through the parameter $\frac{v}{S^{1.50}}$ where S is the plan area of the vehicle.

For example, this restricted analysis does not provide for a limitation on maximum acceleration such as may be dictated by payload characteristics. The value of the vehicle acceleration is easily derived from Equation 14 as

$$\frac{1}{g}\frac{dV}{dt} = \frac{X_s - D}{Mg\left(1 + \frac{gn}{\beta V^2}\right)} = \frac{B(\alpha - 1)}{\left(1 + \frac{gn}{\beta V^2}\right)} \qquad (26)$$

The major drawback to the various applications of this method obviously lies in the validity of the assumptions made concerning constant $\frac{X_s}{W}$, $\frac{D}{W}$, and s.f.c. Usually the assumption of constant s.f.c. is justifiable, but in many cases the parameters $\frac{X_s}{W}$ and $\frac{D}{W}$ vary markedly along the flight path. Nevertheless, suitable mean values usually can be intelligently assessed and utilized in the analysis to give meaningful trends and the gross effects of varying other parameters. It is also possible to assume that the parameters $\frac{X_s}{W}$ and $\frac{D}{W}$ are simple functions of the flight velocity and still obtain relatively simple analytical solutions. However, a considerable loss in generality results.

SECTION IV
CONCLUSIONS

A simple analytical technique has been obtained which permits the relative merit of various engine systems as acceleration power plants to be compared. Subject to the restrictive assumption of constant thrust/drag ratio, the generalized presentation of data enables the quantitative effects of varying engine, vehicle, and trajectory parameters on the payload potential of the vehicle to be evaluated.

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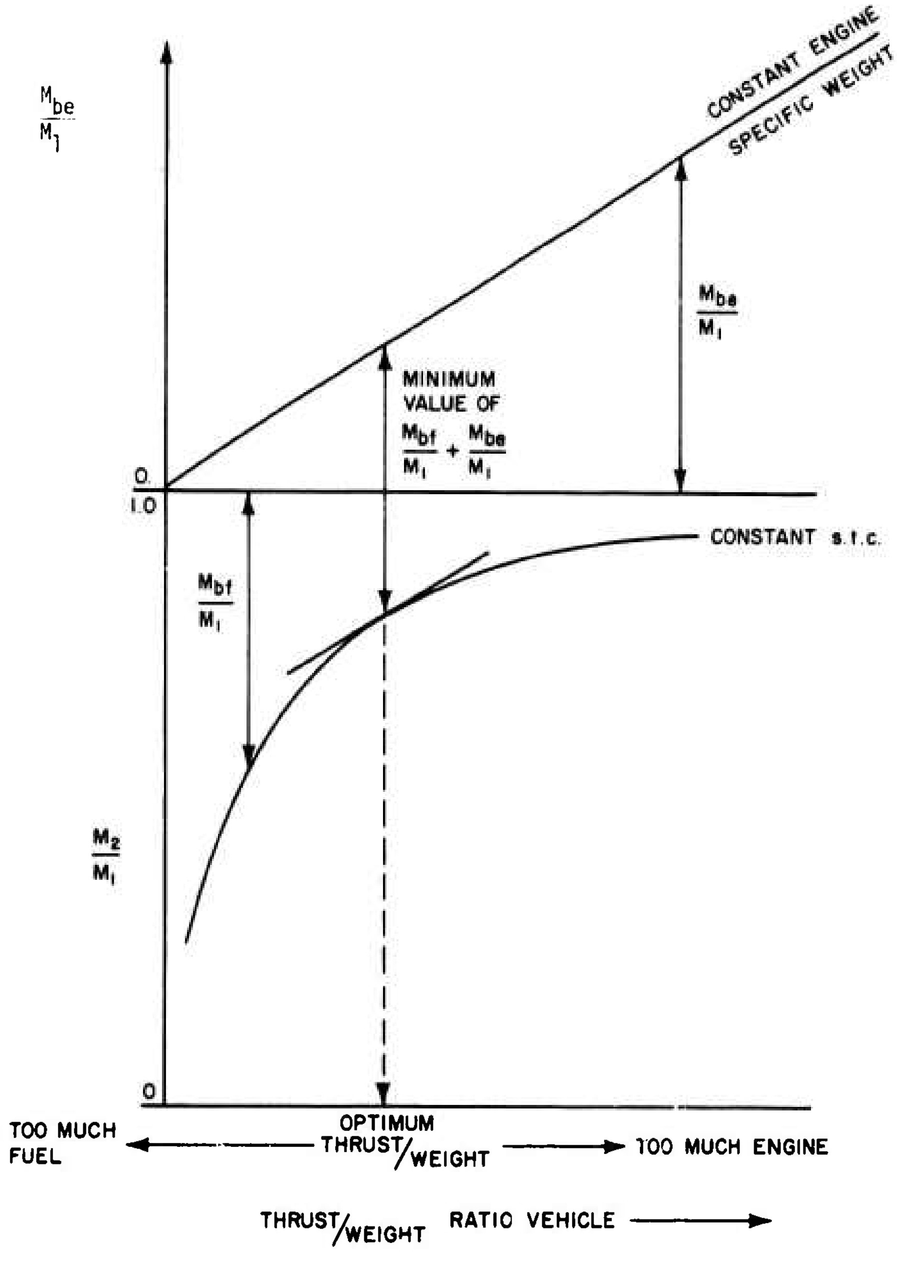

Figure 1. Optimization Process

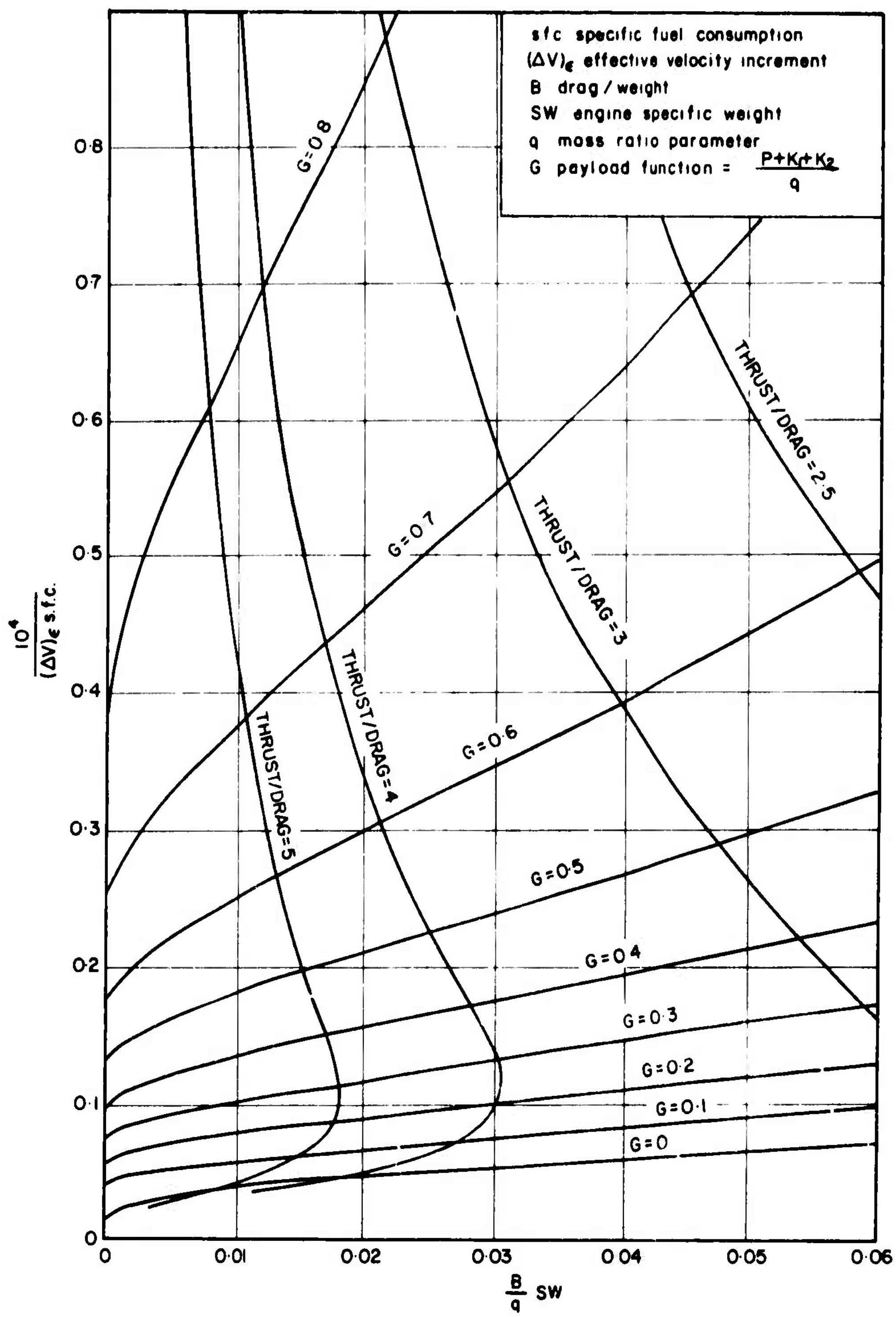

Figure 2. Optimum Conditions for an Acceleration Engine

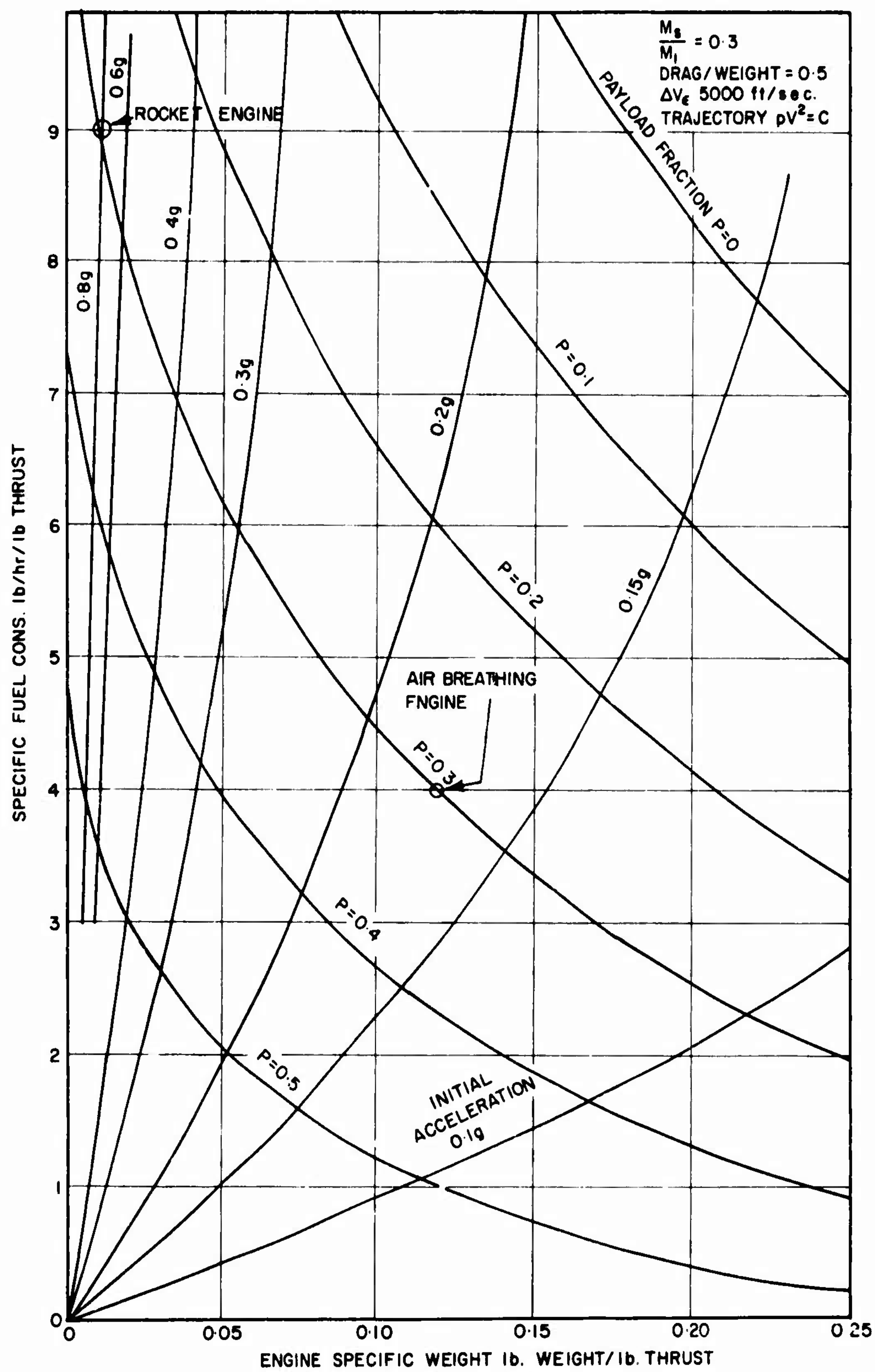

Figure 3. Multistage Vehicle

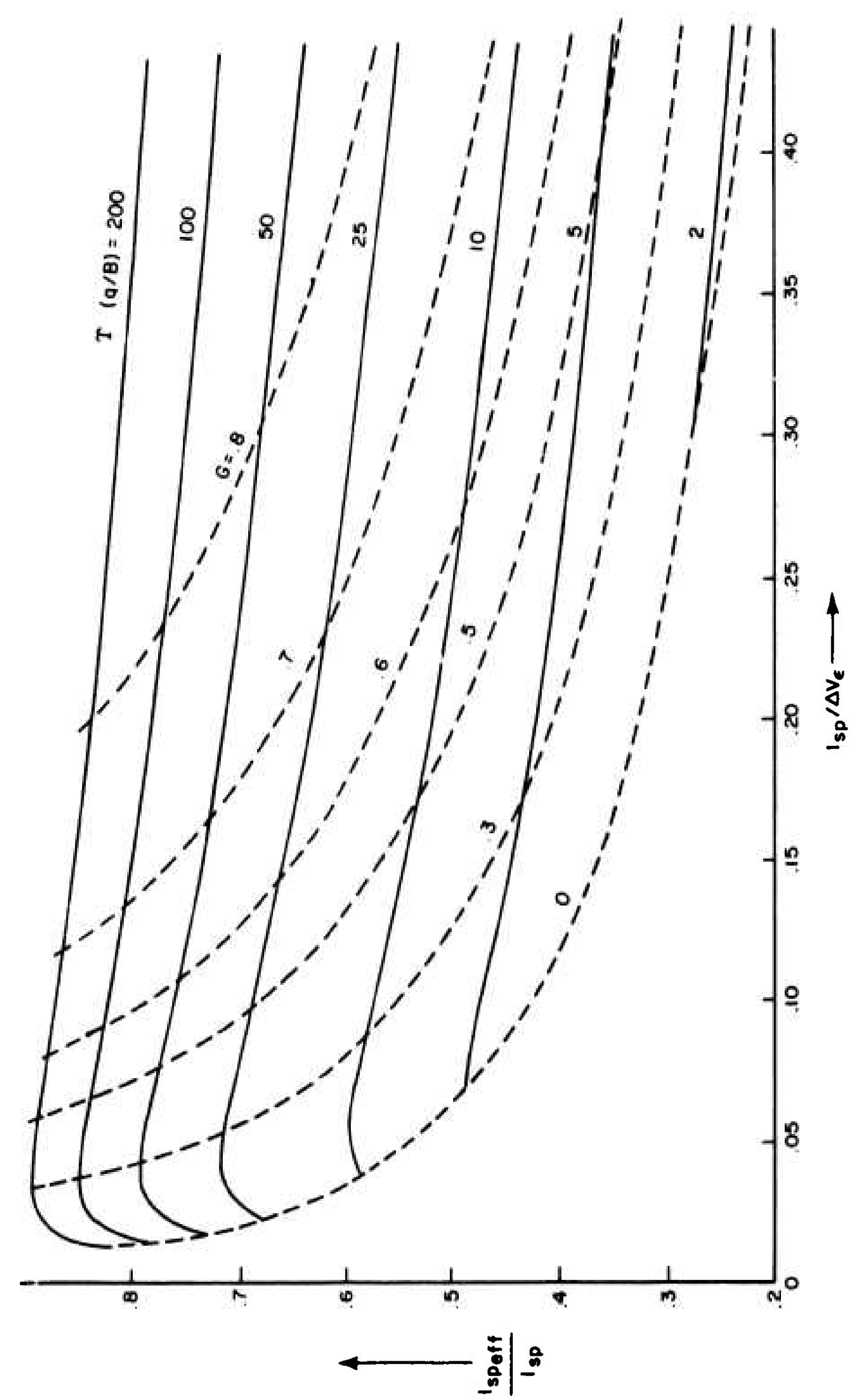

Figure 4. Effective Impulse Achievable

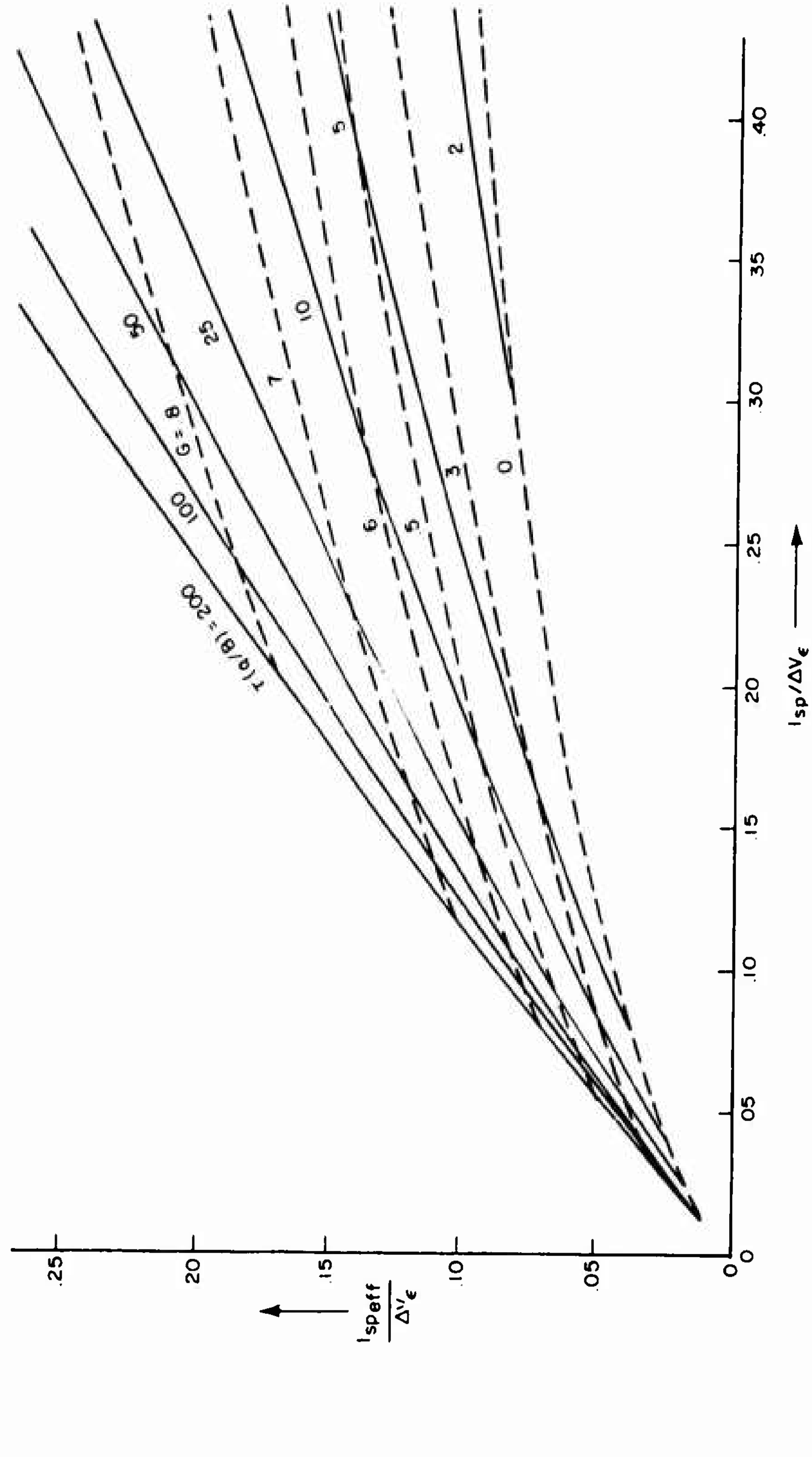

Figure 5. Effective Impulse - Basic Impulse

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